WARDALE ENGINEERING & ASSOCIATES

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CLASS 5AT 4-6-0: FUNDAMENTAL DESIGN CALCULATIONS

1. GENERAL CALCULATIONS.

Note: the general calculations are made for any of the following interesting conditions, as appropriate: starting, maximum drawbar power, maximum indicated power, maximum continuous operating speed and maximum design speed.

1.1. DETERMINATION OF THE TARGET POWER & TRACTIVE EFFORT – SPEED CHARACTERISTICS.

Notes.

1. In practice, different locomotives built to similar standards of thermal design tend to give similar levels of maximum specific indicated power (maximum indicated power per unit of engine mass, $P_{s.ind.max}$) at any given engine rotational speed, n. Therefore it is reasonable at the outset to base the target performance of the 5AT 4-6-0 on the maximum specific indicated power - engine rotational speed characteristic actually achieved with the South African Railways 26 Class 4-8-4 No. 3450, with due allowances for the superior design of the 5AT.
2. The SI system is mostly used. Unless otherwise stated "ton" refers to metric ton of 1000 kg.
3. Numbers in square brackets [ ] in column 2 refer to calculation item numbers in the Fundamental Design Calculations (FDC's): firstly the number identifying the calculations concerned, followed by the item number within those calculations, given in round brackets ( ), e.g. [1.3.(16)] refers to calculations 1.3. item no. (16). Where only a single number is given within square brackets, it refers to an item number within these calculations.
4. ***To save space, unit conversion factors for numerical consistency, where used, are not shown in the calculations. Any apparent small numerical discrepancies are due to giving data to limited places of decimals but to taking the full figure for any calculations involving that data.***
5. References are shown in superscript square brackets [ ] and are given in full at the end of the calculations.
6. Fundamental data is in **bold** type.

Item No.	Item	Unit	Amount				
1	The data in items [2] – [7] applies to SAR 26 Class 4-8-4 No. 3450.						
2	Mass of engine (excluding tender) [1]	ton	123				
3	Actual coupled wheel diameter over tyre rolling plane at time of tests ≈	m	1,50				
4	Speed	km/h	20	40	60	80	100
5	Engine rotational speed, n, = [4] ÷ (π x [3])	Hz	1,18	2,36	3,54	4,72	5,89
6	Maximum indicated power[2]	kW	1 145	2 150	2 915	3 460	3 775
7	Maximum indicated power per unit of engine mass (maximum specific indicated power, $P_{s.ind.max}$) = [6] ÷ [2]	kW/ton	9,31	17,48	23,70	28,13	30,69
8	Maximum design speed, 5AT, [1.3.(27)]	km/h mph	**200** **125**				
9	Coupled wheel diameter, 5AT, [1.3.(22)]	m	**1,88**				
10	Maximum engine rotational speed, 5AT, = [8] ÷ (π x [9])	Hz	9,41				
11	To determine the maximum specific indicated power ($P_{s.ind.max}$) – engine rotational speed (n) curve for 3450 up to the maximum engine rotational speed of the 5AT, the equation linking these two variables is found by iteration from the data of item [7] to be $\{P_{s.ind.max} \approx 8{,}9\ n - 0{,}62\ n^2\}$.						

12	Speed	km/h	20	40	60	80	100	120	130	140	160	180	200
13	Engine rotational speed, 5AT = [12] ÷ (π x [9])	Hz	0,94	1,88	2,82	3,76	4,70	5,64	6,11	6,58	7,53	8,47	9,41
14	$P_{s.ind.max}$ loco 3450 from equation [11]	kW/ton	7,82	14,54	20,17	24,70	28,13	30,47	31,23	31,72	31,86	30,90	28.85
15	Nominal engine mass, 5AT (excluding any ballast applied solely for stability) = [1.3.(6)]	ton	80										
16	[14] x [15]	kW	626	1 163	1 614	1 976	2 250	2 438	2 498	2 538	2 549	2 472	2 308

Item No.	Item	Unit	Amount										
17	The superior design of the 5AT and use of better fuel than SA coal would tend to raise its maximum specific indicated power characteristic above that of 3450. However this would tend to be offset by the 5AT's comparatively small firegrate area and firebox volume relative to overall locomotive size. For the 5AT the figures in item [16] are to be multiplied by a factor (α): the magnitude of α is arrived at by engineering judgement and is speed dependent. Through the middle of the speed range (80 – 140 km/h) α is conservatively set at 1,0 (i.e. the factors given above are presumed to cancel each other). At the top of the speed range (200 km/h), where cut-offs and valve opening times are very short, demanding the utmost in cylinder internal streamlining, α is set at 1,10. At the lower end of the speed range (20 km/h) power is limited by adhesive mass and $P_{s.ind.max}$ is dependent on the ratio of {adhesive mass : total engine mass}.												
18	Ratio of adhesive mass : total engine mass, 3450[1]	-	0,62										
19	Ratio of adhesive mass : total engine mass, 5AT, = [1.3.(9)] ÷ [1.3.(6)]	-	0,75										
20	Factor α at 20 km/h is taken as [19] ÷ [18] ≈ For intermediate speeds α has suitable intermediate values: over the full speed range α is as follows.	-	1,20										
(12)	Speed	km/h	20	40	60	80	100	120	130	140	160	180	200
21	Factor α	-	1,20	1,07	1,02	1,00	1,00	1,00	1,00	1,00	1,01	1,04	1,10
22	**Target 5AT max. indicated power = [16] x [21]**	**kW**	**751**	**1 244**	**1 646**	**1 976**	**2 250**	**2 438**	**2 498**	**2 538**	**2 574**	**2 571**	**2 539**
23	**Target 5AT max. indicated t.e. = [22] ÷ [12]**	**kN**	**135,2**	**112,0**	**98,8**	**88,9**	**81,0**	**73,1**	**69,2**	**65,3**	**57,9**	**51,4**	**45,7**
24	Locomotive rolling resistance is defined as {indicated tractive effort – equivalent drawbar tractive effort}. A specific rolling resistance – speed curve for non-streamlined steam locomotives is derived from the BR Performance & Efficiency Test Bulletins for the BR Standard Class 8 4-6-2[3] and Rebuilt 'Merchant Navy' Class 4-6-2[4], which have similar overall size (including tender) and axle loads (maximum and average) to the 5AT, and which have very similar rolling resistance – speed curves, which are thought to be accurate. Applying these latter curves to the respective total locomotive masses at assumed average mass (i.e. with 2/3 of the total supplies in the tender) gives the specific rolling resistance curves for the two classes concerned[5]. The mean of these two curves is described by an equation of the form $\{r = a + bv + cv^2\}$[6], specifically for the two classes concerned $\{r \approx (45 + 0{,}24v + 0{,}0036v^2)\}$, where r = specific rolling resistance in N/ton and v = speed in km/h. (Note that this equation describes rolling resistance and cannot be used to give starting resistance by putting v = zero (see items [1.3.(164)] and [1.3.(166)].) Hence it is:												
25	Specific rolling resistance, non-streamlined steam locomotive of ≈ 20 ton axle load	N/ton	51,2	60,4	72,4	87,2	105,0	125,6	137,0	149,2	175,6	204,8	237,0
26	Combining data for the DR 03 Class[7], PO 231.722 Class[7], & LNER A4 Class[8], the reduction in locomotive specific rolling resistance due to (average) streamlining can be given by the equation $\Delta r \approx 0{,}0009v^2$, where Δr is in N/ton and v in km/h. As the 5AT will only be semi-streamlined, it is assumed that only half this reduction will apply, i.e. $\Delta r \approx 0{,}00045v^2$. (Note that for very high-speed running (ca. 200 km/h) the need for optimum streamlining is apparent, hence the streamlined form of modern high-speed trains. Semi-streamlining is, however, specified for aesthetic reasons.) Hence Δr is:												
27	Δr	N/ton	0,2	0,7	1,6	2,9	4,5	6,5	7,6	8,8	11,5	14,6	18,0
28	Specific rolling resistance, semi-streamlined steam loco. of ≈ 20 ton axle load = [25] – [27]	N/ton	51,0	59,7	70,8	84,3	100,5	119,1	129,4	140,4	164,1	190,2	219,0
29	For an 80 ton engine with 80 ton tender, average mass of 5AT in service = [1.3.(16)]	ton	142,2										
30	Rolling resistance of 5AT = [28] x [29]	kN	7,3	8,5	10,1	12,0	14,3	16,9	18,4	20,0	23,3	27,0	31,1
31	**Target 5AT maximum drawbar t.e. at constant speed on level tangent track = [23] - [30]**	**kN**	**127,9**	**103,5**	**88,7**	**76,9**	**66,7**	**56,2**	**50,8**	**45,3**	**34,6**	**24,4**	**14,6**

Item No.	Item										Unit	Amount	
(12)	Speed	km/h	20	40	60	80	100	120	130	140	160	180	200
32	**Target 5AT maximum drawbar power at constant speed on level tangent track = [12] x [31]**	**kW**	**711**	**1 150**	**1 478**	**1 709**	**1 853**	**1 875**	**1 834**	**1 760**	**1 539**	**1 221**	**811**
33	Starting indicated tractive effort = [1.3.(53)]										kN	**157**	
34	Nominal starting wheel rim tractive effort = [1.3.(48)]										kN	**146**	
35	Starting drawbar tractive effort on level tangent track = [1.3.(169)]										kN	**134**	
36	Starting drawbar efficiency (= e. db. t.e. ÷ wheel rim t.e.) = [35] ÷ [34] This rather low figure for a fully roller bearing equipped locomotive reflects the large tender mass for the locomotive's nominal tractive effort.										%	92	
37	The tractive effort and power versus speed characteristics are drawn in **Fig. 1.1.1**. Note that the extent of the fall-off in drawbar tractive effort and power at very high speed is influenced by the large tender (required for long operating range) which markedly affects the power : mass ratio of the complete locomotive, the critical factor for very high-speed operation. Thus tender size and the extent of streamlining are two important variables affecting high-speed power.												
38	Adhesion may be a limiting factor on power output. In the steam era adhesion data varied widely and was unreliable, but more recent data can be assumed to be more accurate, although it must be borne in mind that wheel-rail adhesion varies continuously from point to point and adhesion – speed curves give no more than an average picture. To quote from Koffman[9]: "[Adhesion] will depend on wheel and rail conditions, whether wet or dry, on dynamic characteristics of the vehicle, running gear, drive, and upon the geometry of the wheel to rail contact conditions. A reduction of the coefficient of adhesion, μ, is unavoidable even in ideal conditions, neglecting the influence of the above factors, with increasing speed, due to elastic slip." He gives effectively 4 equations for describing the adhesion – speed relationship, which give the following results. (Note: although Koffman is referring to the adhesion of diesel and electric traction, the equations are considered applicable to modern steam traction.)												
39	Speed	km/h	0	20	40	60	80	100	120	140	160	180	200
40	Equation (1): for dry rail: $\mu = \{0,161+7,5/(v + 44)\}$		0,331	0,278	0,250	0,233	0,221	0,213	0,206	0,202	0,198	0,194	0,192
41	Equation (2): for dry rail: $\mu = \{\mu_o(8+0,1v)/(8+0,18v)\}$ where $\mu_o = 0,35$		0,350	0,302	0,276	0,261	0,250	0,242	0,236	0,232	0,228	0,225	0,223
42	Taking the average of [40] and [41] μ for dry rail is:		0,341	0,290	0,263	0,247	0,236	0,228	0,221	0,217	0,213	0,210	0,208
43	Equation (3): for wet rail: $\mu = \{0,129 + 6,0/(v + 44)\}$		0,265	0,222	0,200	0,186	0,177	0,170	0,165	0,162	0,158	0,155	0,154
44	Equation (4): for wet rail: $\mu = \{\mu_o(8+0,1v)/(8+0,18v)\}$ where $\mu_o = 0,26$		0,260	0,224	0,205	0,194	0,186	0,180	0,176	0,172	0,170	0,167	0,165
45	Taking the average of [43] and [44] μ for wet rail is:		0,263	0,223	0,203	0,190	0,182	0,175	0,171	0,167	0,164	0,161	0,160
46	Adhesion has to be related to wheel rim tractive effort: at starting [34] ÷ [33] = [1.3.(42)]:											-	0,93
47	When running the ratio of wheel rim t.e. : indicated t.e. may be expected to be higher than [46] as hydrodynamic lubrication will apply to the piston rings, tail rod bearings, piston rod and tail rod packings, and crosshead slippers, lowering their frictional resistance. Therefore, except at zero speed, for the purposes of these calculations this ratio is made:											-	0,96
48	Wheel rim t.e. = [23] x [47] ([34] at 0 km/h)	kN	146,0	129,8	107,5	94,8	85,3	77,8	70,2	62,7	55,6	49,3	43,9
49	Required value of μ = [48] ÷ [1.3.(9)]	-	0,248	0,221	0,183	0,161	0,145	0,132	0,119	0,107	0,094	0,084	0,075
50	Required value of μ = loco No. 3450	-	0,289	0,266	0,236	0,204	0,181	0,161	-	-	-	-	-

Item No.	Item	Unit	Amount
51	The data of items [42] [45] [49] and [50] is given in **Fig. 1.1.2**. from which it is seen that the adhesion coefficient required for maximum power operation of the 5AT is within that available on both dry and wet rails. However it should be noted that the wet rail curve appears to refer to the maximum possible on wet rails, i.e. when they are very wet. Under conditions such as drizzle, snow, or wet leaves the available adhesion may be considerably less than given by this curve, in which case recourse to sanding will be necessary, especially in the lower half of the speed range. Note that the curves for the 5AT and 3450 are based on the mean tractive effort per coupled wheel revolution, and take no account of variations in tractive effort per revolution. Note also that the 5AT will be in a markedly better position regarding adhesion than 3450, which at full power required more adhesion than was available on wet rails up to some 80 km/h, or over almost its entire permitted speed range.		

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References.

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